Q1

MAGNITUDE

$$(k-5)^2 + (2)^2 + (3k+1)^2 = (6\sqrt{k})^2$$

$$k^2 - 10k + 25 + 4 + 9k^2 + 6k + 1 = 36k$$

$$10k^2 - 40k + 30 = 0$$

$$k^2 - 4k + 3 = 0$$

$$(k-1)(k-3)$$

$$k = 1 \qquad k = 3$$

$$B_1 = (1, -2, 3) \quad \text{OR} \quad B_2 = (3, -2, 9)$$

DISTANCE FROM ORIGIN $= \sqrt{14}$

$$|B_1| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} \quad ✓$$

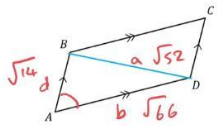
OR

$$|B_2| = \sqrt{3^2 + 2^2 + 9^2} = \sqrt{94} \quad \times$$

$$\boxed{k = 1}$$

Q2

In the parallelogram $ABCD$, AB is parallel to CD, and BC is parallel to AD.



Given that $\overrightarrow{AB} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{AD} = 7\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, find the area of the parallelogram. Give your answer correct to 3 s.f.

$$\text{AREA} = 2 \times ABD = 2 \times \frac{1}{2} bd \sin A$$

$$A = \cos^{-1}\left(\frac{b^2 + d^2 - a^2}{2bd}\right)$$

$$\overrightarrow{BD} = 7\underline{i} - \underline{j} + 4\underline{k} - (3\underline{i} - \underline{j} - 2\underline{k})$$

$$4\underline{i} + 6\underline{k}$$

MAGNITUDE

$$d = |AB| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$$

$$b = |AD| = \sqrt{7^2 + 1^2 + 4^2} = \sqrt{66}$$

$$a = |BD| = \sqrt{4^2 + 6^2} = \sqrt{52}$$

ANGLE A

$$A = \cos^{-1}\left(\frac{14 + 66 - 52}{2\sqrt{14}\sqrt{66}}\right) = 62.576\ldots$$

AREA

$$\sqrt{14}\sqrt{66}\sin(62.576\ldots)$$

$$= 26.981475\ldots$$

$$\boxed{\text{AREA} = 27.0 \text{ units}^2 \text{ (3sf)}}$$

Q3a

The vector $\overrightarrow{RS} = x\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$ makes an angle θ with the positive x-axis.

(a) Show that $x^2 = \frac{90}{\tan^2\theta}$.

[5]

(b) Given that θ is acute and that $\cos\theta = \frac{4}{5}$, find a unit vector in the direction of $\overrightarrow{RS}$.

[4]

$\cos\theta_x = \frac{x}{|RS|}$

$1 - \cos^2\theta = \sin^2\theta$

a) $|RS| = \sqrt{x^2 + 9^2 + 3^2} = \sqrt{x^2 + 90}$

$\cos\theta = \frac{x}{\sqrt{x^2+90}}$

$\cos^2\theta = \frac{x^2}{x^2+90}$

$x^2\cos^2\theta + 90\cos^2\theta = x^2$

$90\cos^2\theta = x^2 - x^2\cos^2\theta$

$90\cos^2\theta = x^2(1 - \cos^2\theta)$

$90\cos^2\theta = x^2\sin^2\theta$

$\frac{90\cos^2\theta}{\sin^2\theta} = x^2$

$x^2 = \frac{90}{\tan^2\theta}$

Q3b

The vector $\overrightarrow{RS} = x\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$ makes an angle θ with the positive x-axis.

(a) Show that $x^2 = \frac{90}{\tan^2\theta}$.

[5]

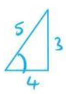
(b) Given that θ is acute and that $\cos\theta = \frac{4}{5}$, find a unit vector in the direction of $\overrightarrow{RS}$.

[4]

IF θ IS ACUTE, x MUST BE POSITIVE

$\cos\theta = \frac{4}{5}$

USE TRIG IDENTITIES OR SIMPLE RATIOS TO FIND $\tan\theta$



b) $\tan\theta = \frac{3}{4}$

FROM (a) $x^2 = \frac{90}{(3/4)^2}$

$x = \frac{4}{3}\sqrt{90} = 4\sqrt{10}$

UNIT VECTOR $\frac{\overrightarrow{RS}}{|R|}$

$|RS| = \sqrt{(4\sqrt{10})^2 + 9^2 + 3^2} = \sqrt{250} = 5\sqrt{10}$

$\frac{1}{5\sqrt{10}}(4\sqrt{10}\,\underline{i} - 9\underline{j} + 3\underline{k})$

Q4

ALL EDGES EQUAL $|AB| = |AC| = |BC| = 162$

$$\sqrt{9^2 + 9^2} = \sqrt{162} \text{ OR } \sqrt{4^2 + 5^2 + 11^2} = \sqrt{162}$$

$$\overrightarrow{AD} = (-2k-1)\underline{i} + 12\underline{j} + (k-1)\underline{k}$$
$$\overrightarrow{BD} = (-2k+8)\underline{i} + 3\underline{j} + (k-1)\underline{k}$$
$$\overrightarrow{CD} = (-2k+3)\underline{i} + 7\underline{j} + (k+10)\underline{k}$$

$$|AD|^2 = (-2k-1)^2 + 12^2 + (k-1)^2 = 162$$
$$4k^2 + 4k + 1 + 144 + k^2 - 2k + 1 = 162$$
$$5k^2 + 2k - 16 = 0$$
$$k = 1.6 \text{ or } k = -2$$

$$|BD|^2 = (-2k+8)^2 + 3^2 + (k-1)^2 = 162$$
$$4k^2 - 32k + 64 + 9 + k^2 - 2k + 1 = 162$$
$$5k^2 - 34k - 88 = 0$$
$$k = 8.8 \text{ or } k = -2$$

$$|CD|^2 = (-2k+3)^2 + 7^2 + (k+10)^2 = 162$$
$$4k^2 - 12k + 9 + 49 + k^2 + 20k + 100 = 162$$
$$5k^2 + 8k - 4 = 0$$
$$k = 0.4 \text{ or } k = -2$$

$$\boxed{D = (4, 13, -2)}$$

Q5

Vectors **a** and **b** are defined by

$\mathbf{a} = pq\mathbf{i} - 24\mathbf{j} + 9r\mathbf{k}$

$\mathbf{b} = 6\mathbf{i} + (p - r)\mathbf{j} + (p - 3q)\mathbf{k}$

Given that $\mathbf{a} = 3\mathbf{b}$, and that $r > 0$, find the values of p, q and r.

[5]

$pq\underline{i} = 18\underline{i}$ $\quad pq = 18$ $\quad p = \frac{18}{q}$ ①

$-24\underline{j} = 3(p-r)\underline{j}$ $\quad -8 = p - r$

$r = p + 8$ ②

$9r\underline{k} = 3(p-3q)\underline{k}$ $\quad 3r = p - 3q$ ③

SUB ② INTO ③

$3(p+8) = p - 3q$

$3p + 24 = p - 3q$

$2p + 3q = -24$

SUB IN ①

$\frac{36}{q} + 3q = -24$

$36 + 3q^2 = -24q$

$3q^2 + 24q + 36 = 0$ $\quad q = -2$ or $q = -6$

$q = -2$ $\quad p = \frac{18}{-2} = -9$ $\quad r = -9 + 8 = -1$

OR

$q = -6$ $\quad p = \frac{18}{-6} = -3$ $\quad r = -3 + 8 = 5$

$r > 0$ $\quad \boxed{p = -3 \quad q = -6 \quad r = 5}$

Q6a

a) $F = ma$ $\quad a = \frac{F}{m}$

$$a = \frac{-3\underline{i} + p\underline{j} + 4\underline{k}}{0.2}$$

$$-15\underline{i} + 5p\underline{j} + 20\underline{k}$$

$$|a|^2 = 15^2 + (5p)^2 + 20^2 = 65^2$$

$$(5p)^2 = 3600$$

$$5p = \pm\sqrt{3600} = \pm 60$$

$$p = \pm\frac{60}{5} = \pm 12$$

$$\boxed{p = \pm 12}$$

Q6b

b) $65 \, ms^{-2} > \frac{5\sqrt{221}}{2} \, ms^{-2}$ [5]

NEW FORCE HAS REDUCED SIZE OF TOTAL FORCE, SO AS $q > 0$ P MUST BE NEGATIVE

$p = -12$

RESULTANT VECTOR

$$a = \frac{-3\underline{i} + (q-12)\underline{j} + 4\underline{k}}{0.2}$$

$$a = -15\underline{i} + 5(q-12)\underline{j} + 20\underline{k}$$

$$|a|^2 = 15^2 + (5(q-12))^2 + 20^2 = \left(\frac{5\sqrt{221}}{2}\right)^2$$

$$5(q-12) = \pm \frac{55}{2}$$

$$q - 12 = \pm \frac{11}{2}$$

$$q = \pm \frac{11}{2} + 12$$

$$\boxed{q = 6.5 \qquad q = 17.5}$$

Q7a

a) $F = ma$

$F_g = -(750 \times 9.8)k = -7350k$

$F_g + T + B + W =$ RESULTANT FORCE

$100\underline{i} - 150\underline{j} - 60\underline{k}$

$$|a| = \frac{\sqrt{100^2 + 150^2 + 60^2}}{750} = \frac{19}{75}$$

$$= 0.2533\dot{3}$$

$$\boxed{0.253 \text{ ms}^{-2} \text{ (3sf)}}$$

Q7b

b) $100\underline{i} - 150\underline{j} - 60\underline{k}$

K IS NEGATIVE SO SUBMARINE IS SINKING

$$\cos\theta_z = \frac{z_F}{|F|} \quad \left(\text{WILL BE SAME AS } \frac{z_a}{|a|}\right)$$

$$\theta_z = \cos^{-1}\left(-\frac{60}{\sqrt{100^2+150^2+60^2}}\right)$$

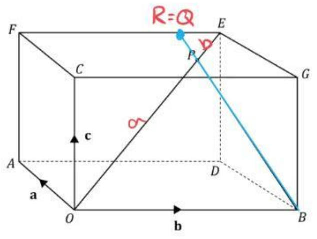
$$= 108.4084\ldots$$

$$= 108.4^\circ \quad 1\text{dp.}$$

$$\boxed{\text{SUBMARINE IS SINKING AT AN ANGLE OF } 108.4^\circ}$$

Q8

The diagram below shows a cuboid whose vertices are O, A, B, C, D, E, F and G.



P is a point on the diagonal OE that divides OE in the ratio $a:b$, where $a > b$.

Show that if line segment BP is extended it will intersect FE, and show that FE is divided in the ratio $(a-b):b$ by the point of intersection.

[11

$\overrightarrow{OE} = \underline{a} + \underline{b} + \underline{c}$ $\quad \overrightarrow{OP} = \lambda\overrightarrow{OE}$

$= \lambda\underline{a} + \lambda\underline{b} + \lambda\underline{c}$

LET Q BE THE POINT ON THE EXTENSION $\overrightarrow{BP}$

LET R BE A POINT ON $\overrightarrow{FE}$

$\overrightarrow{BP} = -\underline{b} + \lambda\overrightarrow{OE} = \lambda\underline{a} + (\lambda-1)\underline{b} + \lambda\underline{c}$

$\lambda = \frac{a}{a+b}$ $\qquad \frac{1}{2} < \lambda < 1$

$\overrightarrow{BQ} = m\overrightarrow{BP}$ $\qquad m > 1$

$\overrightarrow{OQ} = \underline{b} + m\overrightarrow{BP}$

$= m\lambda\underline{a} + (m\lambda - m + 1)\underline{b} + m\lambda\underline{c}$

$\overrightarrow{OR} = \underline{a} + \mu\underline{b} + \underline{c}$ $\qquad 0 < \mu < 1$

IF BP EXTENDS TO INTERSECT FE

$\overrightarrow{OQ} = \overrightarrow{OR}$

$m\lambda = 1$ $\qquad m\lambda - m + 1 = \mu$

$m = \frac{1}{\lambda}$ $\qquad 1 - \frac{1}{\lambda} + 1 = \mu$

$2 - \frac{a+b}{a} = \mu$

$\frac{2a}{a} - \frac{(a+b)}{a} = \mu$

$\mu = \frac{a-b}{a}$

IF $\mu = \frac{a-b}{a}$ AND $0 < \frac{a-b}{a} < 1$

R IS A POINT ON FE MEANING

BP EXTENDS TO INTERSECT FE

μ FRACTION ALONG FE OF POINT R

R DIVIDES FE IN RATIO $\mu : (1-\mu)$

$$\frac{a-b}{a} : 1 - \frac{a-b}{a}$$

$$\frac{a-b}{a} : \frac{\cancel{a}}{a} - \frac{\cancel{a}-b}{a}$$

$$\frac{a-b}{\cancel{a}} : \frac{b}{\cancel{a}} \Rightarrow a-b : b$$

$$\boxed{\text{BP EXTENDS TO INTERSECT FE IN THE RATIO } a-b : b}$$